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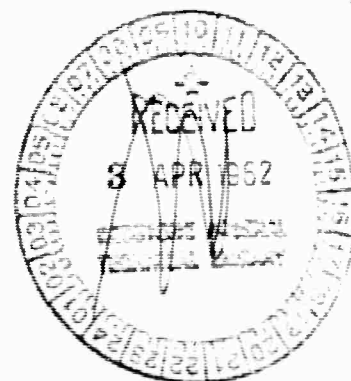# ELECTRO-OPTICAL SYSTEMS, INC.
RESEARCH REPORT

A PROBLEM IN TURBULENT REACTING GAS DYNAMICS - Saul Feldman

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A PROBLEM IN TURBULENT REACTING GAS DYNAMICS - *Saul Feldman*

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ABSTRACT

The case of a streamtube of cold gas going through a strong normal shock wave is treated. Chemical reaction is allowed to occur downstream of the shock, where the pressure and the distribution of certain turbulent correlation functions are prescribed as a function of distance. It is shown that the turbulence modifies the flow field mainly through the effects it has on the reaction rates leading to a hotter gas than if just the mean values of the thermodynamic functions for the turbulent flow were considered. The Gibbs' function for the fluid is also modified by the turbulent fluctuations, thus leading to different final equilibrium conditions.

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1. INTRODUCTION

The present work was motivated by an attempt to find theoretical explanations of the observable phenomena regarding trails left in the atmosphere by objects flying through it at hypersonic speeds.

In the case of equilibrium flow, it has been shown by the author (1)[3] and others (2) that for laminar flow, the trails left by blunt objects could be miles long depending on the detector used by the observer. It was also recognized (1), and recently quantitatively substantiated by Lees and Hromas (3), that when the trails become turbulent, the cooling occurs much more rapidly than in the laminar diffusion case. It is also known that when certain metallic pellets are fired at high speed or when small meteors disintegrate in the atmosphere, they leave luminous trails which sometimes are longer than estimated from the laminar or turbulent thermodynamic equilibrium models (for pure air, for example) studied in the already mentioned references. This effect has been easily "explained away" in the past by saying that the ablation products contaminate the high temperature air, thus leading to large increases in radiation.

Although the foregoing explanation may be the correct one in many physical situations of interest, the question was investigated by the author, sometime ago, regarding the possibility of the existence of a new effect caused by the possible non-linear interaction of the turbulent oscillations (in a compressible reacting fluid) with the reaction rates themselves. Could this interaction affect the rate of cooling of the turbulent wake of a blunt-nosed body? The answer turned out to be in the affirmative, but the effect was a function of the magnitude of the oscillating quantities relative to their mean values. Some of these quantities have been measured in relatively low speed flows, and for those cases, they are of

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[3]Numbers in parentheses denote references at the end of this paper.

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small magnitude. However, Slattery and Clay (4) have recently shown
experimentally that the oscillations in hypervelocity wakes can be
large. It was this last result that motivated the present quantitative
study.

2. EFFECT OF TURBULENCE ON THE CONSERVATION EQUATIONS OF A REACTING FLUID

In order to bring to the forefront the coupling of the turbulence
with the chemical kinetics, we will neglect all normal gradients when
compared with the ones in the direction of motion. Thus, Reynolds and
viscous stresses and laminar and turbulent conduction disappear from the
continuity, momentum and energy equation. In general, this is a very
important restriction, but is not important for the purpose of this paper.
The problem will thus be reduced to the inviscid adiabatic flow in a
streamtube, with a prescribed pressure as a function of distance. However,
the conservation equations, so drastically simplified, contain quantities
which are in general functions of time. When mean values in time are
taken, certain correlation functions appear. The most important effect,
however, occurs in regard to the modification of the chemical kinetic
rates.

2.1 The Conservation Equations

The conservation equations for a compressible reacting fluid
with no normal gradients subject to a prescribed pressure gradient will
be written. The reason for treating essentially the case of a streamtube
which is subject to a given pressure distribution, is that it is the
simplest case that could be considered in which the main features of a
hypersonic wake of a blunt body can be kept in the problem: the strong
pressure gradients which govern the chemical kinetics of the problem.
This streamtube could be interpreted as the first approximation to the
center streamtube of a turbulent reacting wake. In order to keep the
problem simple, the analysis will be restricted to a diatomic gas which
can dissociate and recombine. Let any scalar quantity time varying

quantity $\tilde{q}$ be decomposed into $q+q'$, where q represents the time averaged value and q' is the time dependent part. Also, let bars over a quantity denote a time average taken at a given point in space and will be used only when its deletion may lead to ambiguity. The equations for the mean quantities in one-dimensional flow are given below (5)

Momentum:
$$\rho u \frac{du}{dx} + \overline{\rho'u'} \frac{du}{dx} = -\frac{d}{dx}\left(p + \rho\overline{u'^2} + u\overline{\rho'u'} + \overline{\rho'u'^2} \right)$$
$$[1]$$

where u, p, ρ and T denote, respectively, the time averaged values of velocity, pressure, density and temperature, $\overline{u'^2}$ is the Reynolds stress in the x-direction, s denotes axial distance, non-dimensionalized for example with respect to body radius r_n; i.e. $s = x/r_n$; R is undissociated (cold) gas constant, the primes denote time varying quantities, and the bars denote time averages at a fixed position in space. The subscript ∞ denotes values of the quantities in the undisturbed gas ahead of the projectile. Our streamtube will consist, at the extreme left, of the free-stream gas which first goes through a normal shock wave (discontinuous) across which no chemical reaction takes place. Downstream of the shock wave, chemical reaction is allowed to proceed with a prescribed pressure history which can be approximately prescribed a priori for a particular body shape because, for our purpose, it is not a sufficiently sensitive function of the reaction rates.

Continuity:
$$\frac{A_\infty}{A} = \frac{\rho u}{\rho_\infty u_\infty}\left(1 + \frac{\overline{\rho'u'}}{\rho u} \right)$$
$$[2]$$

where A denotes the cross section of the streamtube, and the subscript ∞ denotes free stream conditions.

<u>Energy</u>: $\dfrac{d}{dx}\left[\rho u\left(h + \dfrac{u^2}{2}\right)\right] + \dfrac{d}{dx}\left[\rho uh\left(\overline{\dfrac{\rho'h'}{\rho h}} + \overline{\dfrac{\rho'u'}{\rho u}} + \overline{\dfrac{u'h'}{uh}} + \overline{\dfrac{\rho'u'h'}{\rho uh}}\right)\right] =$

$$= \dfrac{1}{2}\ \dfrac{d}{dx}\left\{\rho u^3\left[3\left(\overline{\dfrac{\rho'u'}{\rho u}} + \overline{\dfrac{u'^2}{u^2}} + \overline{\dfrac{\rho'u'^2}{\rho u^2}}\right) + \overline{\dfrac{\rho'u'^3}{\rho u^3}} + \overline{\dfrac{u'^3}{u^3}}\right]\right\} \qquad [3]$$

where h denotes enthalpy.

From statistical mechanics, if it is assumed that the vibrational energy is always in equilibrium with the translational energy (i.e. vibrational excitation rates are infinitely fast), and this is not a significant restriction for the present purpose, the enthalpy of a dissociated diatomic gas can be written as a function of temperature and atomic mass fraction y, as

$$\dfrac{h}{RT_\infty} = \dfrac{T}{T_\infty}\left\{\left[\left(5 + \dfrac{\alpha}{T}\right)y + \left(\dfrac{7}{2} + \beta\right)(1-y)\right] + y\left(\dfrac{3}{2} - \beta\right)\overline{\dfrac{y'T'}{yT}}\right.$$

$$\left. - (1-y)\ \beta^2\left[\exp(\theta_v/T)\right]\left[1 + \dfrac{1}{2}\dfrac{\theta_v}{T} - \beta\exp(\theta_v/T)\right]\overline{\dfrac{T'^2}{T^2}}\right\} \qquad [4]$$

where θ_v is the vibrational constant of the molecule and $\alpha = 59,365^\circ$K

$$\beta = \dfrac{\theta_v/T}{\exp(\theta_v/T) - 1}$$

<u>State</u>: $\qquad \dfrac{P/P_\infty}{\dfrac{\rho}{\rho_\infty}\dfrac{T}{T_\infty}} = (1+y)\left(1 + \overline{\dfrac{\rho'T'}{\rho T}}\right) + \overline{\dfrac{\rho'y'}{\rho y}} + \overline{\dfrac{y'T'}{yT}} + \overline{\dfrac{\rho'y'T'}{\rho yT}} \qquad [5]$

<u>Chemical Kinetics</u>: $\qquad \dfrac{u_\infty}{r_n}\dfrac{u}{u_\infty}\dfrac{dy}{ds} = \dfrac{I_d}{W} - \dfrac{I_r}{W^2} \qquad [6]$

where I_d and I_r stand, respectively, for the dissociation and recombination terms in the kinetic equation, and W is the atomic weight of the chemical species. The novel results that will be presented, will depend heavily on Eq. 6 which will be derived in detail in the next section.

The unknowns are A, u, ρ, y and T as functions of s, as well as all the correlation functions. These will have to be specified as a function of s and will be discussed in Section 3.

2.2 The Chemical Kinetic Equations

There is no particular virtue in writing what follows in general form for any diatomic molecule. If done in general, some of the expressions will become too lengthy and cumbersome. Any reader interested in using the present approach for his particular reaction will certainly want to rederive all the expressions on his own. Thus, we will specialize all the following discussion to oxygen as a typical component of air.

Consider the reaction

$$0 + 0 + M \underset{k_d}{\overset{k_r}{\rightleftarrows}} 0_2 + M \qquad [7]$$

where the third body M, could either be 0 or 0_2. The k's are the reaction rate constants. Let a superscript on them indicate the type of third body under consideration, and also let $\boxed{0}$ denote the instantaneous concentration of oxygen atoms in moles/cm^3, the reaction rate can be expressed as

$$\frac{d\left[0\right]}{dt} = - 2k_r^0 \left[0\right]^3 - 2k_r^{0_2} \left[0\right]^2 \left[0_2\right] + 2k_d^0 \left[0_2\right]\left[0\right] + 2k_d^{0_2} \left[0_2\right]^2 \qquad [8]$$

The concentrations can be written in terms of mass fractions as

$$\left[0\right] \quad \frac{moles}{cm^3} = \frac{\rho(gm/cm^3)}{W(gm/mole)} \, y,$$

$$\left[0_2\right] \qquad = \frac{\rho}{2W} (1-y) , \qquad [9]$$

where $W=16$ gm/mole and ρ is the density of the gas mixture.

From Ref. (6)

$$k_d^{O_2} = 9k_d \quad , \quad k_d^{O} = 25k_d \; ,$$

where

$$k_d = 3.6 \times 10^{18} \; T^{-1} \exp\left(-\frac{59,365}{T}\right) \frac{cm^3}{mole \; sec} \; , \tag{10}$$

and T is in ^{O}K. We also know, from theoretical chemical kinetics, within the restriction that a Boltzmann distribution exists at the non-equilibrium temperature, that

$$\frac{k_d}{k_r} = \frac{k_d^{O_2}}{k_r^{O_2}} = \frac{k_d^{O}}{k_r^{O}} = K_c(T) \; , \tag{11}$$

where $K_c(T)$ is the equilibrium constant. For the temperatures that occur in oxygen when flying up to velocities of about 23,000 ft/sec, K_c is given from statistical mechanics by

$$K_c(T) = 2.93 \times 10^{-2} \; T^{1/2} \left(1 - e^{\frac{-2273.8}{T}}\right) \left[\frac{\left(5 + 3 \, e^{\frac{-228}{T}} + e^{\frac{-325.9}{T}}\right)^2}{3 + 2.027 \, e^{\frac{-11,392}{T}}}\right] e^{\frac{-59,365}{T}} \quad \frac{moles}{cm^3} \tag{12}$$

In the range between 3000°K and 8000°K, the above expression can be approximated, with less than 10% error (6) by

$$K_c(T) = 1.2 \times 10^3 \; T^{-1/2} \exp\left(-\frac{59,000}{T}\right) \; \frac{moles}{cm^3}. \tag{13}$$

Thus, from Eqs. 10, 11, and 13

$$k_{r_2}^0 = 9k_r \quad , \quad k_r^0 = 25k_r \quad \text{and}$$

$$k_r = 3 \times 10^{15} \, T^{-1/2} \, \frac{cm^6}{mole^2 \, sec} \, . \qquad \left.\right\} \qquad \boxed{14}$$

Inserting Eqs. 9, 10 and 14 into 8 yields

$$\frac{d\tilde{y}}{dt} = \tilde{k}_d \, \tilde{\rho} \, \frac{1-\tilde{y}}{W} \, (20.5 \, \tilde{y} + 4.5) - \tilde{k}_r \, \frac{\tilde{\rho}^2}{W^2} \, \tilde{y}^2 \, (41\tilde{y} + 9), \qquad \boxed{15}$$

where the tilde has been added to emphasize that the quantities are, in the case of turbulent flow, composed of a time averaged and an oscillating value.

Rewriting Eq. 15 in terms of mean and time dependent quantities gives

$$\frac{dy}{dt} + \frac{dy'}{dt} = \frac{1}{W} \, (k_d + k_d') \, (\rho + \rho')(1-y-y') \left[20.5 \, (y+y') + 4.5 \right]$$

$$- \frac{(k_r + k_r')}{W^2} \, (\rho + \rho')^2 \, (y+y')^2 \left[41(y+y') + 9 \right] \qquad \boxed{16}$$

where we have used the fact that

$$\overline{y'}(t) = 0, \, \overline{\left(\frac{dy'}{dt}\right)} = 0, \quad \text{and} \quad \tilde{\bar{y}}(t) = \bar{\bar{y}}(t) = \bar{y}(t) = y(t).$$

Carrying out the operations in Eq. 16, taking time averages, and considering that the time average of a primed quantity or its derivative is zero, leads to

$$\frac{dy}{dt} = \frac{I_d}{W} - \frac{I_r}{W^2} , \qquad \boxed{17}$$

where

$$I_d = (4.5 + 20.5y)(1-y)(\rho k_d + \overline{\rho' k_d'}) + \bar{\rho}\,\overline{y'k_d'}\,(16-41y) + k_d\overline{\rho'y'}\,(16-41y)$$

$$-20.5\,\rho k_d\,\overline{y'^2} + \overline{\rho'y'k_d'}\,(16-41y) - 20.5(\overline{\rho y'^2 k_d'} + k_d\,\overline{y'^2 \rho'}) - 20.5\,\overline{y'^2 \rho' k_d'},$$

$$\boxed{18}$$

$$I_r = Pk_r\bar{\rho}^2 y^2 + \left[2(Q+9)k_r\bar{\rho}y\,\overline{\rho'y'} + 2P\,\bar{\rho}y^2\overline{\rho'k_r'} + (Q+9)\bar{\rho}^2 y\,\overline{y'k_r'} + Pk_r y^2\overline{\rho'^2} + Qk_r\bar{\rho}^2\overline{y'^2}\right]$$

$$+ \left[(Q+9)k_r y\overline{\rho'^2 y'} + 2Qk_r\bar{\rho}\overline{y'^2\rho'} + 2(Q+9)\bar{\rho}y\overline{\rho'y'k_r'} + Py^2\,\overline{k_r'\rho'^2} + Q\bar{\rho}^2\overline{k_r'y'^2} + 41k_r\bar{\rho}^2\overline{y'^3}\right]$$

$$+ \left[Qk_r\overline{\rho'^2 y'^2} + 2P\bar{\rho}\overline{\rho'^2 y'k_r'} + 2Q\bar{\rho}\overline{y'^2\rho'k_r'} + 41y^2\,\overline{k_r'y'\rho'^2} + 82k_r\bar{\rho}\overline{y'^3\rho'} + 41\bar{\rho}^2\overline{y'^3 k_r'}\right]$$

$$+ \left[Qk_r'\overline{\rho'^2 y'^2} + 82\bar{\rho}\overline{y'^3 k_r'\rho'} + 41k_r\overline{y'^3\rho'^2}\right] + 41\,\overline{k_r'\,\rho'^2 y'^3},$$

$$\boxed{19}$$

$$P = 41y + 9,$$
$$Q = 123y + 9.$$

$$\boxed{20}$$

The first term in Eqs. 18 and 19 is the usual one that appears when the flow is laminar. Without having any experimental information on the behavior of the correlation functions that appear in these equations, it could be conceivable that the term I_d which in laminar flow is the contribution to the net rate due to dissociation may here become a recombination term. A similar comment could be made about I_r. When such a thing happens, the phenomenological approach that led to stating the chemical kinetics in the form of Eq. 8 may require modification. Equation 18 contains up to fourth order correlations while Eq. 19 contains up to the sixth order. It should be noted that in deriving Eqs. 18 and

19 no terms were neglected, and that no restrictive assumption was made regarding the size of primed quantities relative to the corresponding mean values (i.e. ρ'/ρ).

In order to reduce Eqs. 18 and 19 to a tractable form, it will be necessary to derive expressions or evaluate the correlation functions that appear. Very few correlations higher than the second have been made in turbulent fluid mechanics which are applicable to the present problem. If the oscillating quantities are small compared to their respective mean values, correlations higher than the second will not be of importance. In the case of hypervelocity wakes Slattery and Clay (4) show that $\overline{(\rho'^2)}^{1/2}/\rho \simeq 0.5$, which is far from small. However, in order to make the problem tractable, it will be assumed in the present paper that only second order correlations are of importance. After obtaining numerical results based on this assumption, they will be examined and evaluated in terms of it. From Eqs. 18 and 19 the second order correlations required are

$$\overline{y'^2}, \ \overline{\rho'y'}, \ \overline{\rho'k_d'}, \ \overline{y'k_d'}, \ \overline{\rho'k_r'}, \ \text{and} \ \overline{y'k_r'} \ . \qquad \boxed{21}$$

These functions should be expressed in terms of $\overline{\rho'^2}$, $\overline{\rho'T'}$, and $\overline{T'^2}$. It will then be necessary to expand each of the prime quantities in Eq. 21 in ρ' and T', take products and then time averages.

From the perfect gas law

$$p+p' = R(\rho+\rho')(T+T')(1+y+y') \qquad \boxed{22a}$$

or

$$p+p' = R\left[\rho T+(\rho T)'\right](1+y+y') \qquad \boxed{22b}$$

Since the perturbation pressure will depend on the square .. the perturbation velocity, it will be assumed, as a first approximation, that $p' = 0$. A discussion of this point is given by Kistler, with regard to compressible boundary layer work, in Ref. 7, p. 295. Simultaneous measurements of all the oscillating quantities in Eq. 22 have not been made, even when $y+y' = 0$.

Some pressure measurements made in boundary layers by different investi-
gations for various flow conditions are briefly reported by Willmarth
in (Ref. 8, p.112) from which

$$\left(\frac{\overline{p'^2}}{p}\right)^{1/2} \leq 10^{-2} \frac{\gamma}{2} M^2 , \qquad [23]$$

where γ is the ratio of specific heats, and M is the Mach number of
the flow external to the boundary aayer. In order to apply this
result to a turbulent wake, it is thought that the significant Mach
number should be based on the velocity defect across the turbulent core
of the wake. Thus, for a blunt object Eq. 22 will approximately be
reduced to $\left(\overline{p'^2}\right)^{1/2} \Big/ p \leq 10^{-2}$.

Setting p'=0 in Eq. 22a and solving for y' gives

$$y' = \frac{(1+y)\overline{\frac{\rho'T'}{\rho T}} + \overline{\frac{y'T'}{T}} + \overline{\frac{\rho'y'}{\rho}} + \overline{\frac{\rho'y'T'}{\rho T}} - (1+y)\left(\frac{\rho'}{\rho} + \frac{T'}{T} + \frac{\rho'T'}{\rho T}\right)}{1 + \frac{\rho'}{\rho} + \frac{T'}{T} + \frac{\rho'T'}{\rho T}}, \quad [24a]$$

or, if using Eq. 22b

$$y' = \frac{\overline{\frac{(\rho T)'y'}{\rho T}} - (1+y)\frac{(\rho T)'}{\rho T}}{1 + \frac{(\rho T)'}{\rho T}} . \qquad [24b]$$

The time averaged terms that appear in Eqs. 24a and b are of higher
order than the others, and can therefore be neglected. In order to end
up with oscillating quantities of a single variable in each term, and
since the oscillating products are such that are not any easier to
measure than the single quantities, we will expand the last term in
parenthesis and the denominator of Eq. 24a into a power series of
ρ'/ρ and T'/T. When the same is done for k_d' and k_r' the results are

$$y' = -(1+y)\left[\frac{\rho'}{\rho} + \frac{T'}{T} + 0\ (\frac{\rho'T'}{\rho T})\right]$$

$$k_d' = k_d\left[(\frac{\alpha}{T} - m)\ \frac{T'}{T} + 0\ (\frac{T'^2}{T^2})\right],\ \text{and} \qquad \boxed{25}$$

$$k_r' = k_r\left[-n\ \frac{T'}{T} + 0\ (\frac{T'^2}{T^2})\right].$$

Taking the products necessary to construct the expressions in Eq. 21, and averaging in time yields

$$\overline{y'^2} = (1+y)^2\left(\overline{\frac{\rho'^2}{\rho^2}} + 2\ \overline{\frac{\rho'T'}{\rho T}} + \overline{\frac{T'^2}{T^2}}\right) + \text{triple correlations}$$

$$\overline{\rho'y'} = -(1+y)\ \rho\left(\overline{\frac{\rho'^2}{\rho^2}} + \overline{\frac{\rho'T'}{\rho T}}\right) + \text{triple correlations}$$

$$\overline{\rho'k_d'} = \rho k_d\ q\ \overline{\frac{\rho'T'}{\rho T}} + \text{triple correlations}$$

$$\overline{y'k_d'} = -(1+y)\ k_d\ q\left(\overline{\frac{\rho'T'}{\rho T}} + \overline{\frac{T'^2}{T^2}}\right) + \text{triple correlations} \qquad \boxed{26}$$

$$\overline{\rho'k_r'} = -\rho k_r\ n\ \overline{\frac{\rho'T'}{\rho T}} + \text{triple correlations}$$

$$\overline{y'k_r'} = (1+y)\ k_r\ n\left(\overline{\frac{\rho'T'}{\rho T}} + \overline{\frac{T'^2}{T^2}}\right) + \text{triple correlations}$$

$$q = \frac{59,365}{T} - 1$$

Inserting Eqs. 26 into Eqs. 18 and 19 yields, after dropping third and higher order correlations,

$$I_d = \rho_\infty \frac{\rho}{\rho_\infty} \, k_d \left[f_1 - f_2 \frac{\overline{T'^2}}{T^2} - f_3 \frac{\overline{\rho'^2}}{\rho^2} + f_4 \frac{\overline{\rho'T'}}{\rho T} \right], \qquad [27]$$

where

$$f_1 = (4.5 + 20.5y)(1-y),$$

$$f_2 = (1+y)\left[q(16-41y) + 20.5(1+y) \right],$$

$$f_3 = (1+y)(36.5 - 20.5y),$$

$$f_4 = (4.5 + 20.5y)(1-y) \, q-(1+y)\left[q(16-41y) + 57 \right], \qquad [28]$$

and

$$I_r = \rho_\infty^2 \left(\frac{\rho}{\rho_\infty} \right)^2 k_r \left[g_1 + g_2 \frac{\overline{T'^2}}{T^2} + g_3 \frac{\overline{\rho'^2}}{\rho^2} + g_4 \frac{\overline{\rho'T'}}{\rho T} \right] \qquad [29]$$

where

$$g_1 = (41y + 9)y^2,$$

$$g_2 = 3(1+y)\left[\frac{1}{2}(41y+6)y + (41y+3)(1+y) \right],$$

$$g_3 = y\left[(41y+9)y+3(1+y)\left[(41y+3)\frac{(1+y)}{y} - 2(41y-6) \right] \right], \qquad [30]$$

$$g_4 = -\frac{9}{2}(41y+6)(1+y)y+2\left[3\,(41y+3)(1+y)^2 - \frac{1}{2}(41y+9)y^2 \right].$$

It should be pointed out that f_1 and g_1, in Eqs. 28 and 30, are the usual terms that appear in the chemical kinetic equations when there is no turbulence. The remaining f's and g's can be interpreted as influence coefficients of the correlation functions on the reaction rates.

We will discuss next how to arrive at some reasonable values for

the correlation functions necessary to solve the problem.

3. ESTIMATES OF THE CORRELATION FUNCTIONS BASED ON SOME EXPERIMENTAL DATA

There are many different correlation functions that appeared in the foregoing discussion. Only few of them have ever been measured regardless of the fluid mechanical problem under discussion. Therefore, based on whatever experimental data are available, we will have to at least evaluate the order of magnitude of the functions involved. In order to do this in a reasonable manner, we will quickly and superficially review in what follows some of the typical values of certain correlation functions that have been measured in different flow problems, i.e. boundary layers, jets and wakes.

3.1 Measurements in the Boundary Layer

Typical measurements are those presented by Klebanoff(9) of the $\left(\overline{u'^2}\right)^{1/2}\big/u \leq 10^{-1}$ in incompressible boundary layers; those of Kistler(7) of $\left(\overline{T'^2}\right)^{1/2}\big/T \approx 10^{-1}$, $\left(\overline{u'^2}\right)^{1/2}\big/u \leq 9\times10^{-2}$, $\dfrac{\overline{T'u'}}{Tu} \approx -7 \times 10^{-3}$, $\overline{(\rho u)'}\big/\rho u \leq 0.14$ in compressible boundary layers between mach numbers of 1.72 and 4.67 where the stagnation temperature was 300°K.

3.2 Measurements in Turbulent Jets

Data for incompressible jets can be found in Townsend's book (10). The maximum disturbances occur near the center of the jet and gives $\left(\overline{u'^2}\right)^{1/2}\big/u \leq 0.15$, $\overline{u'v'}/u^2 \leq .008$. Corrsin and Uberoi (11) measured for a heated jet $\left(\overline{T'^2}\right)^{1/2}\,T \leq 10^{-1}$.

3.3 Measurements in Wakes

3.3.1 Incompressible Wakes

For incompressible wakes flows Townsend shows that $\left(\overline{u'^2}\right)^{1/2}\big/u \leq .04$, $-\overline{u'v}/u^2 \leq .006$.

3.3.2 Compressible Wakes

Demetriades (12) measured $\overline{(\rho u)'}/\rho u \leqslant 0.40$ in the wake of a
cylinder at free stream mach number of 5.8 and Reynolds numbers between
18,500 and 48,500. Slattery and Clay (4) have taken schlieren photographs
of the wake left behind a spherical projectile of 1/2 inch in diameter
flying at 8370 ft/sec in air at a pressure of 30 mm of mercury. They
took densitometer tracings of the negative at several portions downstream
of the projectile and derived from them the density fluctuations as given
in the following table:

x/d	10	50	400	800
$\left(\overline{\rho'^2}\right)^{1/2}/\rho$	0.05	0.25	0.90	0.50

The fluctuations in density can be seen to be quite large, and they are the
first experimentally derived values obtained at hypervelocities. These
values should be compared with $\overline{\rho'u'}/\rho u \leqslant 0.4$ obtained by Demetriades (11).
Most of the oscillation of the mass flow correlation must be due to the
oscillation in density, because if it were due to the oscillation in
velocity, the kinetic energy invested in the turbulence would be pro-
hibitively large. Before use is made of a high value for the density
fluctuation, an attempt will next be made of explaining why such a value,
although high, could be justified in the case of hypersonic wakes.

Consider the wake of a hypervelocity sphere (Fig. 1) that
has a turbulent wake. Lees and Hromas (3) have analyzed, in some detail,
the behavior of such a turbulent wake in equilibrium flow. Within
the turbulent core, one would expect to have eddies of various tempera-
tures. The range of temperature variation of the eddies would be
between the low temperature in the inviscid flow at the turbulent edge
and the highest value in the turbulent core. Hromas (13) has given the
author some of his calculations for a turbulent trail of a sphere of 1/2
inch in diameter, flying at sea level at 9500 ft/sec. At this velocity
there are no chemical reactions and the enthalpy on the center line h_o, of the
trail divided by the edge value h_f, is equal to the corresponding
temperature ratio, T_o/T_f.

For a blunt body, close to the wake's neck, T_o/T_f should be near unity; very far downstream, when all the gas has cooled, $T_o/T_f \to 1$. In between, the outer inviscid wake cools rapidly and this ratio should have a peak. Fig. 2 shows this ratio for two bodies under different flight conditions.

We will use the 9500 ft/sec curve to arrive at some conclusions regarding the turbulence structure necessary to make possible a value of $\left(\overline{\rho'^2}\right)^{1/2}/\tilde{\rho} \approx 0.5$, as obtained by Slattery and Clay (4). Let us assume that in the turbulent core we have a mixture composed of lumps of hot and cold gas, which if no radial gradients of pressure allowed, represent spots of low and high density gas. Let ρ_c, ρ_h and ρ represent respectively the density of the cold gas, the hot gas and the mean value. We will assume that the structure is periodic in space with a period of length L. We will neglect that the gas may be reacting chemically. Thus

$$\frac{\rho_c}{\rho_h} = \frac{T_h}{T_c} = \tau_1 \qquad\qquad [31]$$

The magnitude of the temperature T_c for the cold gas would not be lower (although it may be higher) than T_f, the temperature at the edge of the turbulent wake. T_h would be higher than the mean temperature calculated, for example, by Lees and Hromas. Thus, τ_1 would be approximately given by

$$\tau_1 = T_o/T_f \qquad\qquad [32]$$

Let the value of the square of the density fluctuation be given by

$$\frac{\overline{\rho'^2}}{\rho^2} = \tau_2 , \qquad\qquad [33]$$

where the total density $\tilde{\rho}$ is given by $\tilde{\rho} = \rho + \rho'$, and the density structure is assumed to be as shown in Fig. 3. For that structure, the mean
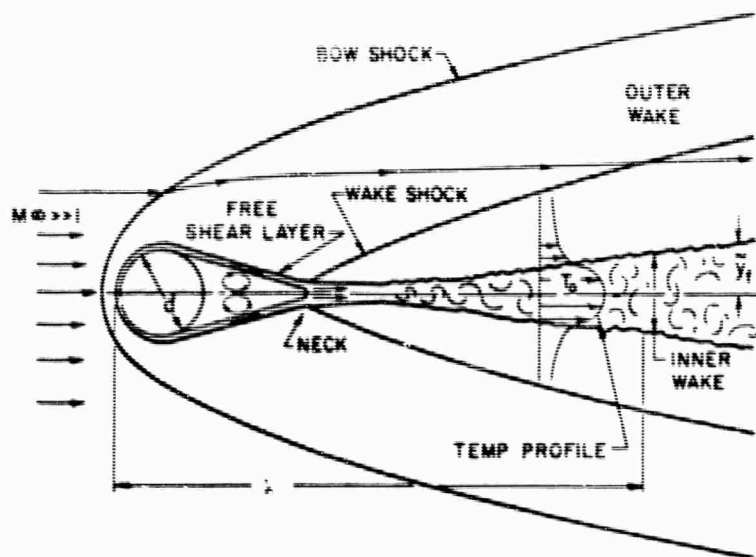
FIG. 1

WAKE BEHIND BLUNT BODY AT
HYPERSONIC SPEEDS (Ref. 3)
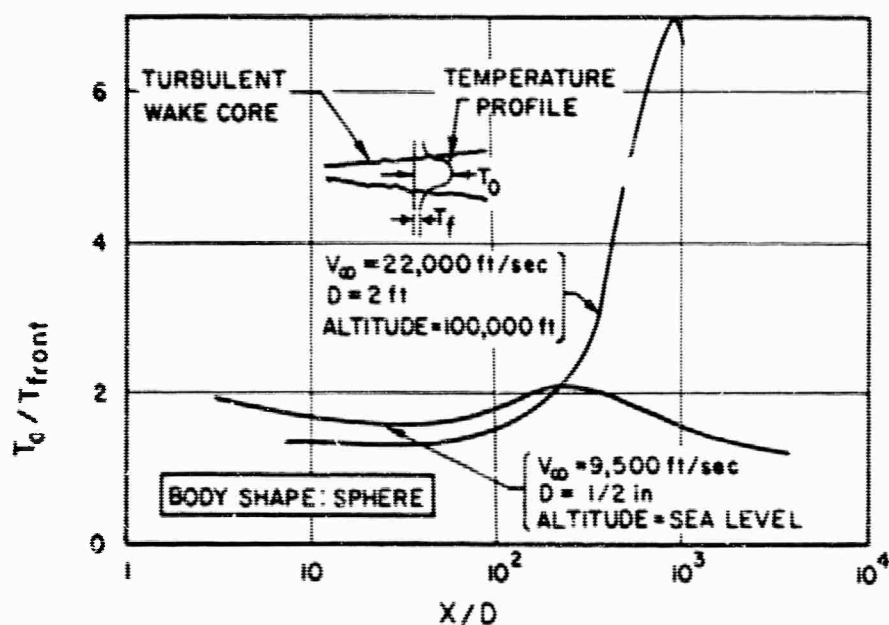
FIG. 2

TEMPERATURE RATIO OF CENTERLINE -
TO EDGE VALUES FOR TURBULENT WAKES
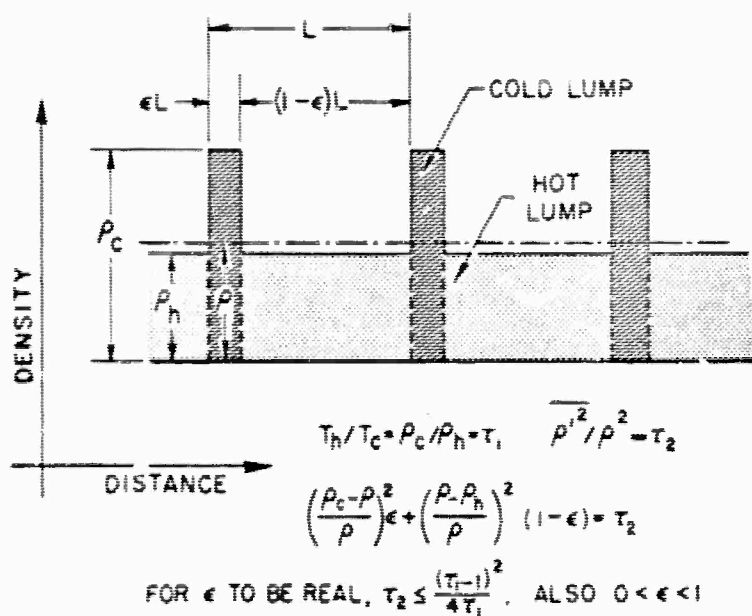
FIG. 3

ASSUMED DISTRIBUTION OF DENSITY
IN A "TURBULENT" GAS

$$T_h/T_c = \rho_c/\rho_h = \tau_1 \qquad \overline{\rho'^2}/\rho^2 = \tau_2$$

$$\left(\frac{\rho_c - \rho}{\rho}\right)^2 \epsilon + \left(\frac{\rho - \rho_h}{\rho}\right)^2 (1-\epsilon) = \tau_2$$

FOR ϵ TO BE REAL, $\tau_2 \leq \frac{(\tau_1 - 1)^2}{4\tau_1}$. ALSO $0 < \epsilon < 1$

density is given by

$$(\rho_c - \rho)\,\epsilon = (\rho - \rho_h)\,(1-\epsilon),$$

or

$$\frac{\rho}{\rho_h} = 1 + (\tau_1 - 1)\,\epsilon, \tag{34}$$

where ϵ is defined in Fig. 3.

Interpreting Eq. 34 for the structure of Fig. 3, gives

$$\left(\frac{\rho_c - \rho}{\rho}\right)^2 \epsilon + \left(\frac{\rho - \rho_h}{\rho}\right)^2 (1-\epsilon) = \tau_2,$$

which can be rewritten as

$$\left[\left(\tau_1 \frac{\rho_h}{\rho} - 1\right)^2 - \left(1 - \frac{\rho_h}{\rho}\right)^2\right]\epsilon + \left(1 - \frac{\rho_h}{\rho}\right)^2 = \tau_2. \tag{35}$$

We would like to ask now, that if one specifies a value of τ_1 (obtained from Lees and Hromas's calculations) and τ_2 (obtained from Slattery and Clay's measurements), is there any hope to find a value of ϵ, from Eqs. 34 and 35, that would make physical sense for the model of Fig. 3. Physical sense means that the value of ϵ satisfies

$$0 < \epsilon < 1 \tag{36}$$

and even more, ϵ should be small compared to unity; that is, only less than half of the mixture should be cold gas.

If the value of ρ_h/ρ from Eq. 34 is used in Eq. 35, solving for ϵ leads to

$$\epsilon = \frac{-(2\tau_2 - w)\,w \pm \sqrt{(2\tau_2 - w)^2\,w^2 - 4\,(\tau_2 + 1)\,\tau_2 w^2}}{2\,(\tau_2 + 1)\,w^2}, \tag{37}$$

where

$$w = \tau_1 - 1. \tag{38}$$

In order for ϵ to be real, the discriminant has to be positive or zero. Thus,

$$(2\tau_2 - w)^2 - 4(\tau_2 + 1)\tau_2 \geqslant 0,$$

$$\text{or} \qquad \tau_2 \leqslant \frac{(\tau_1 - 1)^2}{4\tau_1} \qquad\qquad \boxed{39}$$

It should be noted that for $\tau_1 \gg 1$, $\tau_2 \leqslant \tau_1/4$ or $\left(\overline{\rho'^2}\right)^{1/2}/\rho \leqslant \sqrt{\tau_1}/2$.

In the present case (Fig. 2) at 800 diameters downstream from the pellet at 9500 ft/sec, $\tau_1 = 1.66$, thus giving from (39) $\tau_2 \leqslant 0.0658$. Slattery and Clay give $\left(\overline{\rho'^2}\right)^{1/2}/\rho \approx 0.5$ which is equivalent to $\tau_2 = 0.25$. Thus, our model of Fig. 3 cannot be even approximately correct if Slattery and Clay's measurement is right. Although the details of the model of Fig. 3 are certainly incorrect when compared with real turbulence, the magnitude of the mean values we compute from it cannot be radically wrong. Thus, it would be interesting to determine the value of ϵ that Eq. 37 would yield if τ_2 were less than 0.0658. Let, for example, $\tau_2 = 0.04$, which corresponds to $\left(\overline{\rho'^2}\right)^{1/2}/\rho = 0.2$ instead of 0.5. From Eq. 37 two values of ϵ can be found: either $\epsilon = 0.68$ or $\epsilon = 0.15$. These values lead, from Eq. 34 respectively to either $\rho/\rho_h = 1.41$ or $\rho/\rho_h = 1.09$. Thus, according to the previous discussion, we will choose $\epsilon = 0.15$ and $\rho/\rho_h = 1.09$ as a physically reasonable combination. We, therefore, conclude that either Slattery and Clay's measured value is too high in order to be physically realizable, or that Lees and Hromas's (3) calculations don't give the true temperature ratio in the turbulent wake.

In order to further examine whether the idealization of Fig. 3 leads to meaningful results, we have used the data of Fig. 2 for the 22,000 ft/sec, 2 ft. sphere about 600 diameter downstream, at which point $\tau_1 = 6$. From Eq. 39

$$\sqrt{\tau_2} = \left(\overline{\rho'^2}\right)^{1/2}/\rho \leqslant 1.02. \qquad\qquad \boxed{40}$$

Thus, taking $\tau_2 = 1$ and using Eqs. 37 and 34 yields

$$\begin{aligned}\epsilon_1 &= 0.16, & \rho_1 &= 1.64, \\ \epsilon_2 &= 0.08, & \rho_2 &= 1.32.\end{aligned} \qquad\qquad \boxed{41}$$

where the subscripts 1 and 2 denote the two possible solutions of the problem. Both values of ϵ and ρ, particularly the second ones, are reasonable because such values should be physically realizable.

4. FURTHER SIMPLIFICATION OF THE EQUATIONS FOR A TURBULENT REACTING GAS FLOW.

Before using the conservation equation to do numerical calculations, it will be necessary to specify the correlation functions that appear in Eqs. 1 thru 6. In order for these equations to be consistent with Eq. 6 (where the correlations higher than the second were assumed small), the last term in the right hand member of Eqs. 1 and 5 should be left out, as well as the last three in the right hand member of Eq. 3. The last term of the left hand member of 3 should also be left out. Since little is known about the functional dependence of the correlation functions, their derivatives with respect to x (or s) could be left out. When this is done, the equations reduce to

Momentum:
$$\frac{dV}{ds} = \frac{\dfrac{RT_{\infty}}{u_{\infty}^2}\ \dfrac{d(\rho/\rho_{\infty})}{ds}}{V(\rho/\rho_{\infty})(1+\rho'u'/\rho u)}\quad,\qquad [42]$$

where
$$V = u/u_{\infty},\qquad [43]$$

Continuity:
$$\frac{A_{\infty}}{A} = \frac{\rho u}{\rho_{\infty} u_{\infty}}\left(1 + \overline{\frac{\rho'u'}{\rho u}}\right),\qquad [44]$$

Energy:
$$\frac{h}{RT_{\infty}} = \frac{u_{\infty}^2}{2RT_{\infty}}\left[1 - \left(\frac{u}{u_{\infty}}\right)^2\right] + \frac{h_{\infty}}{RT_{\infty}}\quad,\qquad [45]$$

Statistical Mechanics:
$$\frac{h}{RT_{\infty}} = \frac{T}{T_{\infty}}\left\{\left[\left(5 + \frac{\Omega}{T}\right)y + \left(\frac{7}{2} + \beta\right)(1-y)\right] + y\left(\frac{3}{2} - \beta\right)\frac{\overline{y'T'}}{yT}\right.$$
$$\left. - (1-y)\beta^2\left[\exp(\theta_v/T)\right]\left[1 + \frac{1}{2}\ \frac{\theta_v}{T} - \beta\ \exp(\theta_v/T)\right]\frac{\overline{T'^2}}{T^2}\right\}$$
$$[46]$$

where

$$\frac{\overline{y'T'}}{yT} = -\frac{1+y}{y}\left(\overline{\frac{\rho'T'}{\rho T}} + \overline{\frac{T'^2}{T^2}}\right) \ , \qquad \boxed{47}$$

and

$$\beta = \frac{\theta_v/T}{\left[\exp\left(\theta_v/T\right)\right] - 1} \ , \qquad \boxed{48}$$

State:

$$\frac{\rho}{\rho_\infty} = \frac{p}{p_\infty}\left\{\left(\frac{T}{T_\infty}\right)\left[(1+y)\left(1 + \overline{\frac{\rho'T'}{\rho T}}\right) + \overline{\frac{\rho'y'}{\rho y}} + \overline{\frac{y'T'}{yT}}\right]\right\}^{-1} \qquad \boxed{49}$$

Chemical
Kinetics:

$$\frac{u_\infty}{r_n} v \frac{dy}{ds} = \frac{I_d}{W} - \frac{I_r}{W^2} \ . \qquad \boxed{50}$$

The magnitude of the term $\overline{\rho'u'}/\rho u$ in Eqs. 42 and 44 could be, according to the data presented in Section 3.1, of the order of 0.15. Thus, neglecting such a term in those equations would introduce an error of about 15% in the momentum equation.

From an inspection of the other foregoing equations and Eqs. 27 thru 30, it can be concluded that the reaction rate equations are much more sensitive, for a given value of the correlation functions, than the other conservation equations. Thus, as a first attempt at obtaining some numerical results, the correlation functions were only included in Eq. 50. After numerical results were thus obtained, a check was made of the importance of some of the terms left out, which will be discussed later.

When the correlations are neglected, Eqs. 45 and 46 can be equated and solved for y

$$y = \frac{\dfrac{u_\infty^2 \left(1 - v^2\right)}{2RT_\infty} + \dfrac{h_\infty}{RT_\infty} - \dfrac{T}{T_\infty}\left(3.5 + \dfrac{\theta_v/T}{\exp \theta_v/T - 1}\right)}{\dfrac{T}{T_\infty}\left(1.5 + \dfrac{59365}{T} - \dfrac{\theta_v/T}{\exp \theta_v/T - 1}\right)}. \qquad [51]$$

Differentiation of Eq. 51 with respect to s, insertion of Eqs. 42 and 50 lead to

$$\frac{dT}{ds} = \frac{T_\infty\left[\dfrac{d(p/p_\infty)}{ds}\Big/ p/p_\infty\right] - \left[1.5\,T + 59{,}365 - \dfrac{2274}{(\exp 2274/T)-1}\right]\left(\dfrac{t_n}{u_\infty v}\right)\left(\dfrac{I_1}{w} - \dfrac{I_2}{w^2}\right)}{1.5y + 3.5 + (1-y)\left(\dfrac{2274}{T}\right)^2\left[\dfrac{\exp 2274/T}{\left[(\exp 2274/T)-1\right]^2}\right]}$$

$$[52]$$

In order to solve the differential equations 42 and 52 it is necessary to prescribe the pressure on the streamtube as a function of s. Since we are interested in a blunt body, we will prescribe a Newtonian pressure distribution on the nose as

$$\frac{p}{p_\infty} = \frac{p_s}{p_\infty} \cos^2 s, \qquad 0 \le s \le 0.96 (= 55^\circ) \qquad [53]$$

where the subscript s denotes stagnation conditions. Let p_{sh} denote pressure at 90° away from the stagnation point, then the pressure can be expressed (14) as

$$\frac{p}{p_\infty} = \left[\frac{1}{1 + 2(s-\frac{\pi}{2})}\right]\frac{p_{sh}}{p_\infty} + \left[\frac{1}{1 + \dfrac{1}{2(s-\frac{\pi}{2})}}\right]^{p_{sh}/p_\infty}, \quad s \ge \frac{\pi}{2} \qquad [54]$$

where p_{sh} can be determined (15) from

$$\frac{p_{sh}}{p_\infty} = 1 + \left(0.045 - \frac{0.827}{M_\infty^2}\right) \frac{p_s}{p_\infty} , \qquad \boxed{55}$$

where M_∞ is the free stream Mach number. Between 55° and 90° the pressure can be fitted with an expression of the form

$$\frac{p}{p_\infty} = as^2 + bs + c , \qquad 0.96 \leqslant s \leqslant \frac{\pi}{2} \qquad \boxed{56}$$

where

$$a = 1.2823 \frac{p_s}{p_\infty} - 16.379 \frac{p_{sh}}{p_\infty} ,$$

$$b = -3.7837 \frac{p_s}{p_\infty} + 43.090 \frac{p_{sh}}{p_\infty} , \qquad \boxed{57}$$

$$c = 2.7795 \frac{p_s}{p_\infty} - 26.271 \frac{p_{sh}}{p_\infty} .$$

The necessary pressure derivatives are

$$\frac{d(p/p_\infty)}{ds} = -\frac{p_s}{p_\infty} \sin 2s , \qquad\qquad 0 \leqslant s \leqslant 0.96 \quad \boxed{58}$$

$$= 2a\,s + b , \qquad\qquad 0.96 \leqslant s \leqslant \frac{\pi}{2} \quad \boxed{59}$$

$$= \frac{2\,p_{sh}/p_\infty}{\left[1+2\left(s-\frac{\pi}{2}\right)\right]^2} \left\{ \left[\frac{2\left(s-\frac{\pi}{2}\right)}{1+2\left(s-\frac{\pi}{2}\right)}\right]^{(p_{sh}/p_\infty - 1)} - 1 \right\} \quad s > \frac{\pi}{2}$$

$$\boxed{60}$$

Finally, the correlation functions necessary in Eqs. 27 and 29 are $\overline{T'^2}/T^2$, $\overline{\rho'^2}/\rho^2$ and $\overline{\rho'T'}/\rho T$. The calculations will be made for the case where the streamtube has no turbulence from s=0 to s=3, it builds up linearly with distance until it reaches s=4 (i.e. in one radius) to a value that stays constant with s (Fig. 4), i.e.

$$\overline{T'^2}/T^2 = \overline{\rho'^2}/\rho^2 = \overline{\rho'T'}/\rho T = 0 \qquad \text{for } 0 \leqslant s \leqslant 3 \qquad \boxed{61}$$

$$\left.\begin{aligned}
\frac{\overline{T'^2}}{T^2} &= a_1 \; \frac{s-s_1}{s_2-s_1} \;, \\[2ex]
\frac{\overline{\rho'^2}}{\rho^2} &= a_2 \; \frac{s-s_1}{s_2-s_1} \;, \\[2ex]
\frac{\overline{\rho'T'}}{\rho T} &= a_3 \; \frac{s-s_1}{s_2-s_1} \;,
\end{aligned}\right\} \qquad s_1 \leqslant s \leqslant s_2 \qquad \boxed{62}$$

where $s_1 = 3$ and $s_2 = 4$,

$$\overline{T'^2}/T^2 = a_1, \quad \overline{\rho'^2}/\rho^2 = a_2 \text{ and } \overline{\rho'T'}/\rho T = a_3 \qquad s > s_2 \qquad \boxed{63}$$

5. DISCUSSION OF NUMERICAL RESULTS.

Before deciding what values of a_1, a_2 and a_3 should have in the calculations, it is important to realize some of the restrictions that have to be imposed.

Only a_2 has been measured (4). Thus the other a's have to be conjectured. a_1 and a_2 are positive definite. In a non-reacting gas, γ = constant and if in Eq. 22 a ρ' = 0, $\rho'T'$ would always be a negative quantity, and further more would be zero. However, in our calculations $\overline{\rho'T'}$ was assumed to be negative or zero. No calculations were made for

$\overline{\rho'T'}$ positive. It also follows, from the non-reacting case, that $a_1 = a_2$. Various other relative values of a_1 and a_2 were used also.

A physically impossible combination of a's manifests itself in an obvious way by I_d becoming negative. I_d or I_r both have to be positive at all times.

If the a's are kept smaller than 10^{-2}, the present problem as treated herein is approximately correct. For larger values of the a's the triple and higher order correlation terms, which have been left in the calculation, become important. These could be kept, but the difficulty is to prescribe reasonable values to them in a particular calculation, since not only have they not been measured, but it may be impossible to do so.

Fig. 4 shows the temperature T and atomic oxygen mass fraction y, along a streamtube as a function of downstream distance measured from the normal shock non-dimensionalized with respect to nose radius. Two spheres have been used, one of 1/2 inch radius and the other of 5 inches, flying at 20,000 ft/sec in an oxygen atmosphere where density is 10^{-2} the normal value. When $x/r_n = 0$, the temperature reaches a high value while $y = 0$. After dissociation the temperature drops and y builds up. Fig. 4 shows the history of the flow for $s > 10^{-2}$. It can be seen that for large values of s, after the pressure has long ago reached the free-stream value, the variables reach a constant equilibrium value. The reason for the equilibrium state for each sphere is not the same is due to different entropies of the flow in each case: for the smaller sphere the flow is further out of equilibrium during the expansion process, thus leading to a larger entropy in the final state, which represents a higher final temperature.

It should be noted that the final equilibrium state will be affected by the presence of turbulence. This can be clearly seen from the reaction rate equations when they go to equilibrium. One other way of looking at this fact is that the turbulence terms modify Gibbs' function, which when minimized, leads to a different equilibrium condition.

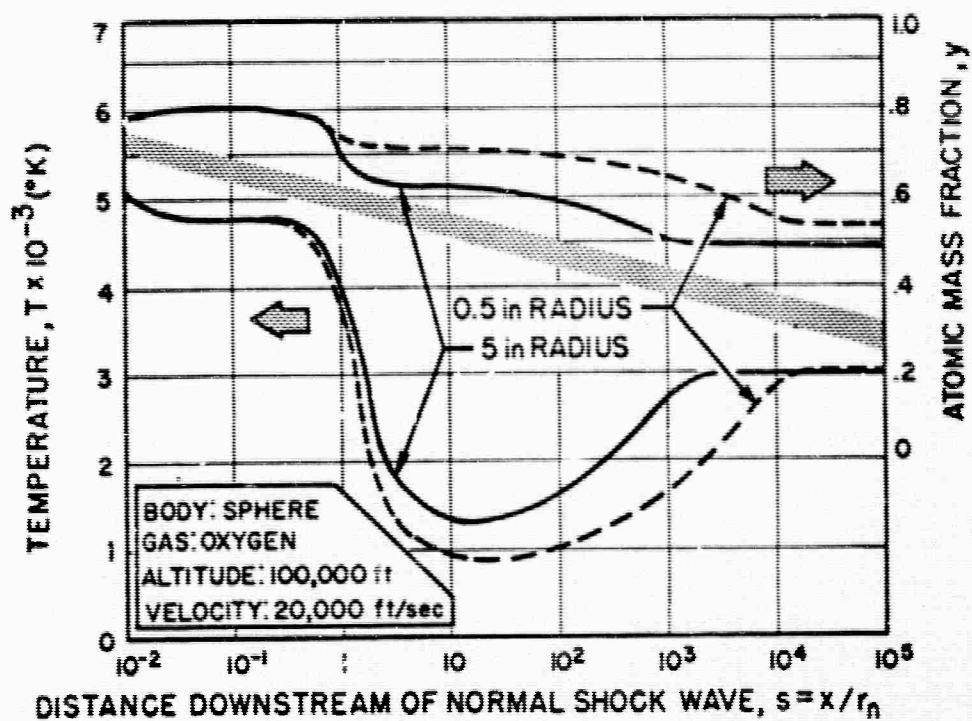Figs. 5 through 7 present some of the results obtained for the case

FIG. 4 TEMPERATURE AND ATOM MASS FRACTION DISTRIBU-
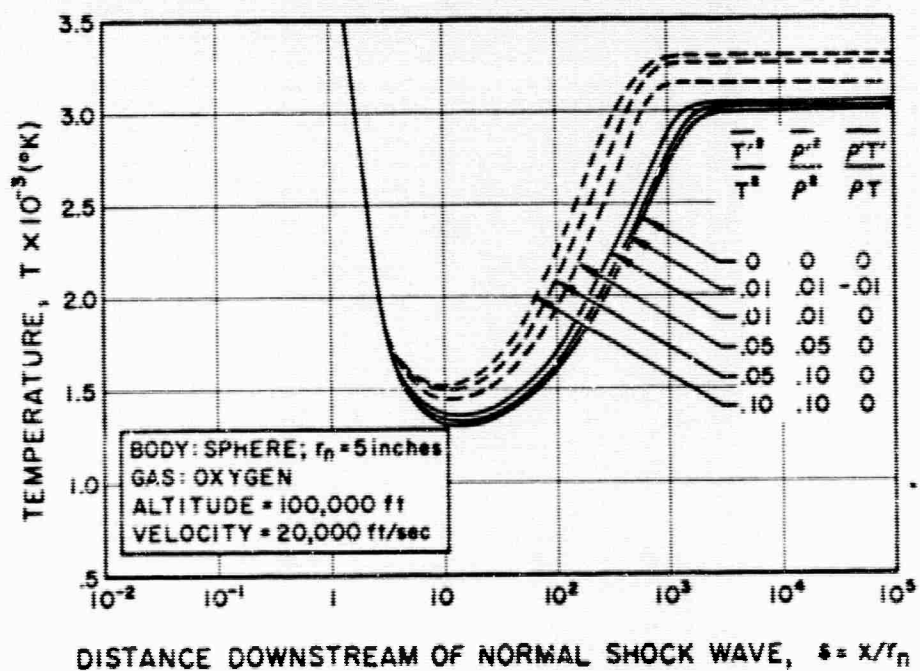TION ON AXIAL STREAMTUBE. LAMINAR FLOW FOR
TWO SPHERES.

FIG. 5 TEMPERATURE DISTRIBUTION ON AXIAL STREAMTUBE.
A COMPARISON BETWEEN LAMINAR AND TURBULENT
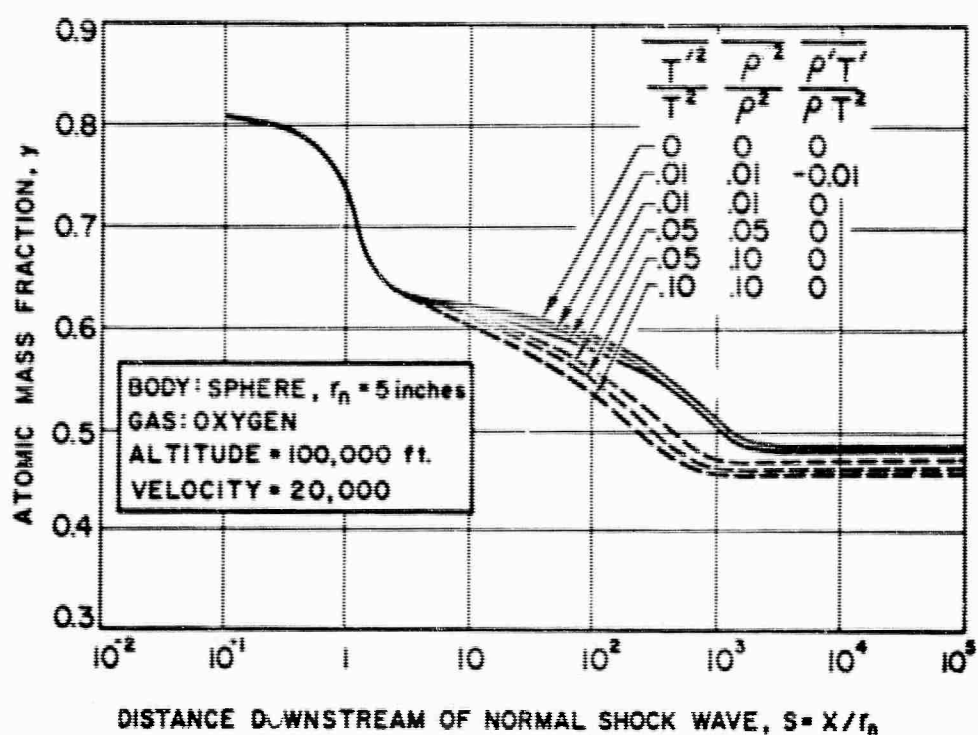FLOW. SPHERE OF 5 INCH RADIUS.

FIG. 6   ATOM MASS FRACTION DISTRIBUTION ON AXIAL
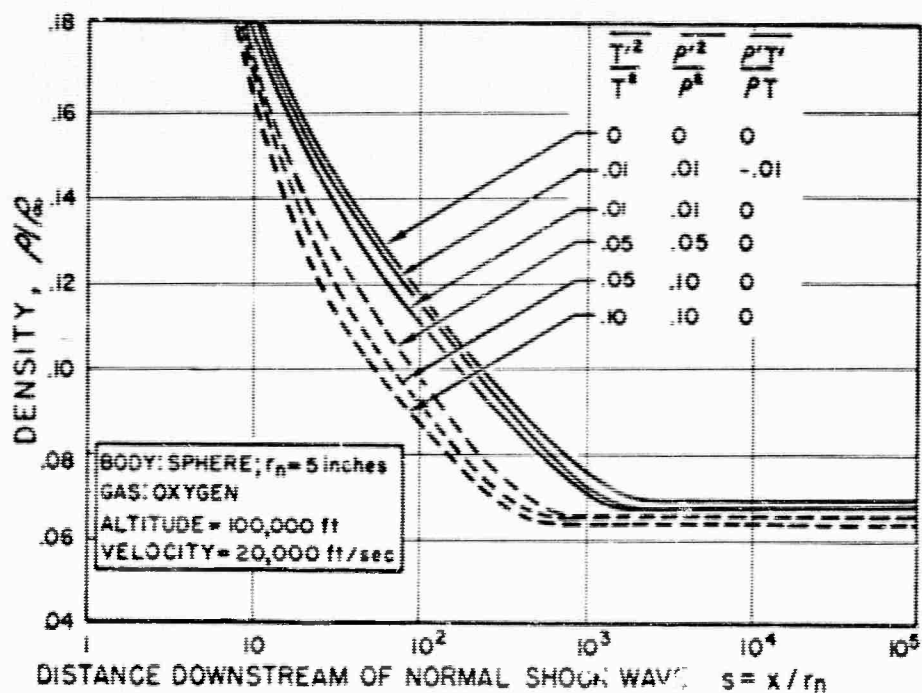   STREAMLINE. SPHERE OF 5 INCH RADIUS.

FIG. 7   DENSITY RATIO DISTRIBUTION ON AXIAL
          STREAMTUBE. SPHERE OF 5 INCH RADIUS

r_n = 5 inches. In these figures the curves should be compared with the non-turbulent results. The solid curves are to be trusted, while the dashed ones cannot be proven to be correct. However, the fact that they form a reasonable family is meaningful. In Fig. 5 for example it should be noted that although the final equilibrium temperature is increased by at most 300°K when the flow is turbulent, between s = 10^3, the increase can be near 1000°K. This hotter wake could then lead to significantly more radiation. Using the data of these figures, it is possible to calculate the absolute magnitude of the radiation for the different assumed situations. Such results would be more sensitive to the turbulence than the temperature variations.

Figs. 8 through 10 give similar results for the case where the sphere nose radius is 1/2 inch. It can be shown that if the correlations neglected in Eqs. 42 to 49 were included in the calculations, higher temperatures would ensue.

6. CONCLUDING REMARKS

From the foregoing results it can be said that turbulent chemical reacting flows under the conditions studied can lead to flow fields, which at any given station are of a higher temperature than without turbulence. The fact that, in hypervelocity wakes, the mean values of the turbulent correlations can be large, leads to the possibility of significant effects, although we have not proven this with all the necessary rigor. The surface has just been scratched and much work remains to be done.

FIG. 8

TEMPERATURE DISTRIBUTION ON
AXIAL STREAMTUBE. A comparison
between laminar and turbulent
flow. Sphere of 0.5 inch radius.

FIG. 9

ATOM MASS FRACTION DISTRIBUTION
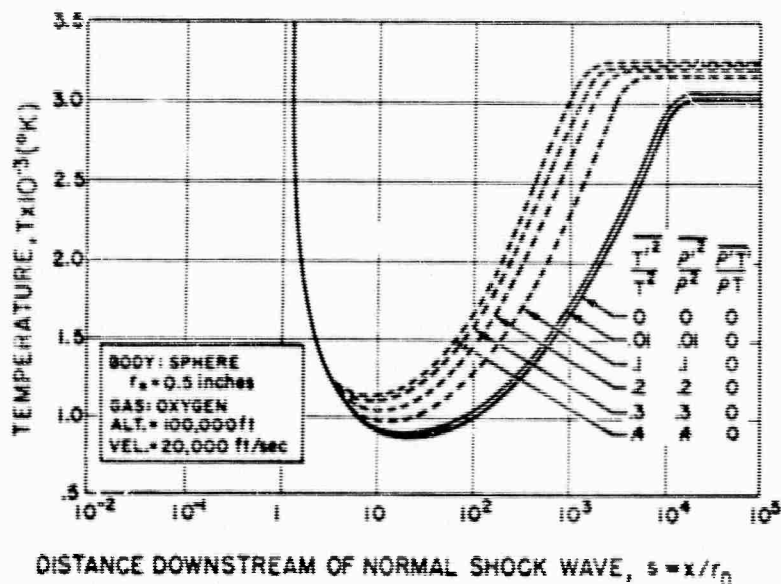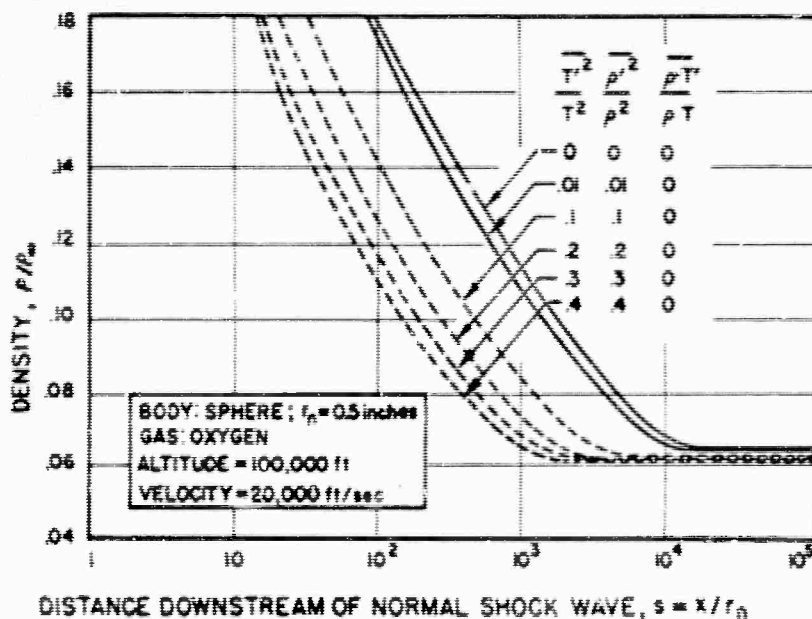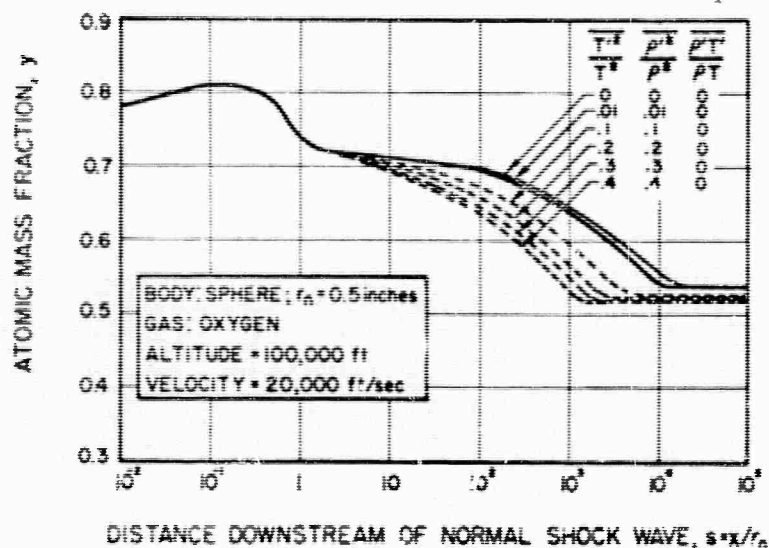ON AXIAL STREAMLINE. SPHERE OF
0.5 INCH RADIUS.

FIG. 10

DENSITY RATIO DISTRIBUTION ON
AXIAL STREAMTUBE. SPHERE OF
0.5 INCH RADIUS.

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